Improving Paratransit Scheduling by Accounting for Dynamic and Stochastic Variations in Travel Time

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Travel times in an urban traffic environment often are subject to dynamic and stochastic variations due to random fluctuations in travel demands, frequent interruptions of traffic controls, and unpredictable occurrences of traffic incidents. Although these variations inevitably affect the real-life performance of a paratransit system, they have not been taken into account in the routing and scheduling process by most existing paratransit scheduling systems. The potential effects of these variations on the operational characteristics of a paratransit system such as vehicle productivity and schedule reliability are examined. A dial-a-ride routing and scheduling system capable of modeling dynamic and stochastic travel times was used in the analysis. A series of numerical experiments was performed on a practical problem from the city of Edmonton, Alberta, under hypothetical travel time variation patterns. It was found that both dynamic and stochastic variations in travel times had important effects on the quality of the schedules, and an appropriate consideration of these variations in the scheduling process could substantially improve the reliability and productivity of the schedules.

The potential for improving the productivity and reliability of dial-a-ride paratransit, also called demand responsive transit, has significantly increased in recent years because of the latest advances in information technologies such as automated vehicle location systems (AVL), digital communications, and computers (*1–3*). The applications of these technologies in transportation engineering, broadly labeled as intelligent transportation systems (ITS), will make a large amount of real-time data, such as current vehicle locations, traffic conditions, and customer requests, available for use in the paratransit management and operation process. It is expected that the effective use of these data in the design of paratransit services will yield more productive and more reliable routes and schedules. However, most existing computerized paratransit scheduling systems have not yet integrated the functionality required to take advantage of this increased data availability (*4,5*). The intention of this report is to take the application of one of these kinds of data—travel times—as an example to illustrate the importance of adequate data use and the need for improved scheduling systems.

In a dial-a-ride paratransit system, the major role of a scheduling system is to determine the pickup and drop-off routes and times for a fleet of vehicles carrying customers between specified origins and destinations. The underlying problem is referred to as the "dial-a-ride problem" (DARP). To model and solve the DARP, the essential information needed includes the travel times between origin locations and destinations (or O-D travel times). Because of the limitation in data availability and computation processing ability, the DARP historically has been modeled in a static and deterministic manner in the sense that O-D travel times are assumed to be constants. In an urban traffic environment, however, O-D travel times may be highly time dependent (or dynamic) and stochastic because of the random fluctuations of travel demands, frequent interruptions of traffic controls, and unpredictable occurrences of traffic incidents. It can be expected that, in situations of high uncertainty, the service vehicles may not be able to follow the established schedules based on the traditional models, and thus a reliable service may not be guaranteed. For example, on the basis of the assumption of deterministic O-D travel times, it would be feasible to schedule a vehicle to drop off a customer at the destination at the most desired drop-off time. However, the actual drop-off time will not be exactly the scheduled drop-off time because of the randomness of the vehicle's travel time. The most obvious drawback associated with the assumption of constant O-D travel times is that it may result in erroneous and inefficient schedules. Although all these problems are theoretically true, issues such as what the actual consequences of these variations would be, what factors would influence the schedules, have yet to be addressed. Therefore, the objective of this report is to investigate whether there are meaningful differences between considering and not considering the dynamic and stochastic variations in travel times in the performance of schedules created by a routing and scheduling system.

METHODOLOGY

Problem Statement and Overview

The DARP is defined to construct a set of feasible and efficient routes and schedules to satisfy transportation requests (trips) made by the system clients. A trip specifies the number of persons of each specific type to be transported, a pick-up location and a drop-off location, and the desired pick-up or drop-off time or both. Two types of clients are considered: the ambulatory, who can use regular seats, and clients who must remain seated in wheelchairs. A fleet of vehicles that can accommodate these seating requirements is available to operate the routes. The travel time between any locations (or O-D travel time) in the service area is given or can be calculated on the basis of other available information such as location coordinates, average travel speed, and road networks. The dwell times at pickup and drop-off locations for each trip are known. It should be noted that these dwell times can be incorporated easily into the O-D travel time and therefore will be considered as part of the O-D travel time without further clarification.

The DARP can be classified further into two types: the *static* DARP, which needs to be solved at the beginning of every operational day with all the trip requests known in advance (for example,

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booked 1 day in advance, or subscribed for regular service), and the *real-time* DARP, in which the objective is to determine the assignment of new trips into the existing schedules of vehicles in real time. This study deals only with the static DARP.

The DARP commonly is formulated to minimize a general objective function (or cost function) with a set of service quality constraints (*6,7*). In this paper, the cost function is defined as a weighted sum of the total client inconvenience (or disutility), as measured in terms of *excess ride time* (the difference between the scheduled ride time and the ride time without diversions for other customers) and *service time deviation* (the difference between the scheduled pickup and drop-off times and their most desired pickup and drop-off times), and the cost to the service providers, as measured in terms of the total vehicle travel time and the number of vehicles used.

The service quality constraints specify that the ride time of each client must be less than a maximum allowable ride time and that all clients must be picked up (dropped off) after (before) their most desired pickup (drop-off) times with service time deviations less than a maximum allowable value. Note that the latter defines a time interval, or *service time window*, during which the service must take place, as shown in Figure 1(*a*).

When the O-D travel times are modeled as random variables, the cost function and service constraints need to be redefined. Compared to a deterministic model, the main difference in modeling is the use of probabilistic service time windows, as illustrated in Figure 1(*b*). In the probabilistic model, the service time constraint specifies that the probability of the arrival time within the desired time window must be greater than a prespecified threshold value called *minimum reliability*. For example, if a minimum reliability of 90 percent is used in scheduling, all clients must be scheduled to be picked up or dropped off during their service time windows with at least a 90 percent chance. More detailed descriptions on the modeling methodology can be found elsewhere (*8,9*).

Several variations of the DARP can be defined on the basis of how O-D travel times are modeled (*8*). As shown in Figure 2(*a*), the constant O-D time model (DARP-C) assumes that the travel time between each O-D pair is deterministic and independent of the time of day. In the DARP-S, O-D travel times are modeled as random, but time-independent, variables [Figure 2(*b*)]. The third class of the DARP, DARP-D, assumes that the O-D travel times are dynamic, that is, time-dependent but not stochastic, as shown in Figure 2(*c*). When the travel times are modeled as both dynamic and stochastic, as shown in Figure 2(*d*), the result is the last class of problems—DARP-DS. Note that the problem DARP-DS is the most general model among all the classes, and thus a solution algorithm to solve the DARP-DS also can be used to solve other classes of the DARP. The following section presents an overview of the solution procedure used to solve the DARP-DS.

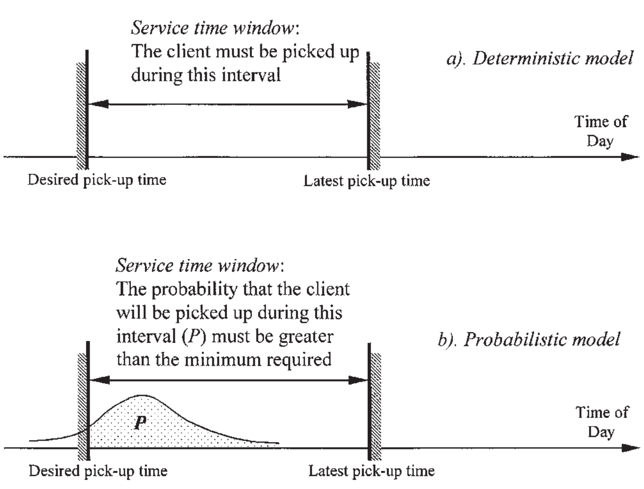

FIGURE 1 Service time window for client specifying desired pickup time: deterministic versus probabilistic models.

Solution Method

Since the early 1970s, several computer algorithms and programs have been developed to solve the DARP (*6,10–12*). However, one of the major disadvantages of these algorithms is that they are based on the assumption of constant O-D travel times and thus essentially are applicable to the DARP-C. Recently, the author developed a new routing and scheduling system called FirstWin that allows the dynamic and stochastic nature of the origin-destination (O-D) travel times to be modeled explicitly (*13*). This system will be used in the following analysis to solve the various versions of the DARP discussed previously. This section briefly discusses the algorithm included in FirstWin to solve the DARP-DS. A more detailed description of the FirstWin software is provided by Fu and Teply (*8*).

It is well known that the DARP is computationally intractable, and only heuristic algorithms are feasible to solve its real-life instances. One of the most widely used heuristic routing and scheduling procedures is called the insertion algorithm (*10*). The algorithm processes customer trips sequentially and attempts to insert one trip at a time into the available vehicles. This algorithm has been modified to solve the DARP-DS with a new objective function and a set of probabilistic constraints. The modified insertion algorithm has the following procedure:

1. Determine the pickup and drop-off time windows for all trips based on their desired service times, the maximum allowable service time deviation, and the maximum allowable ride time.
2. Select the trip $i = 1$ from the trip list.
3. Select the vehicle $k = 1$ from the fleet.
4. Examine all possible ways in which trip i can be inserted into the partial route of vehicle k and find the insertion that results in a minimum insertion cost (or objective function value). For each insertion, the feasibility of the route after the insertion is first verified, ensuring that the vehicle's capacity is not exceeded at each stop along the route, that all clients are picked up or dropped off within their service time windows at a probability greater than the minimum reliability, and that the mean ride time of each trip is less than the maximum allowable ride time. An optimal schedule for the new route then is determined by using an algorithm similar to the one presented by Dumas et al. (*14*). Note that the dynamic characteristics of the travel times impose a need to update both the means and the variances of the travel times between stops and of the arrival times at individual stops after each insertion. It is assumed that the O-D travel times are independent and normally distributed with known mean and variance. Consequently, the estimation of the arrival time variances at individual stops on a given route can be determined readily. If all vehicles are examined, then go to the next step; otherwise, select the next vehicle $k + 1$ and repeat this step.
5. If it is not feasible to assign trip i to any vehicle, then set trip i as a "leftover" trip. Otherwise, assign trip i to vehicle $k*$, for which the insertion cost is minimal among all the vehicles.

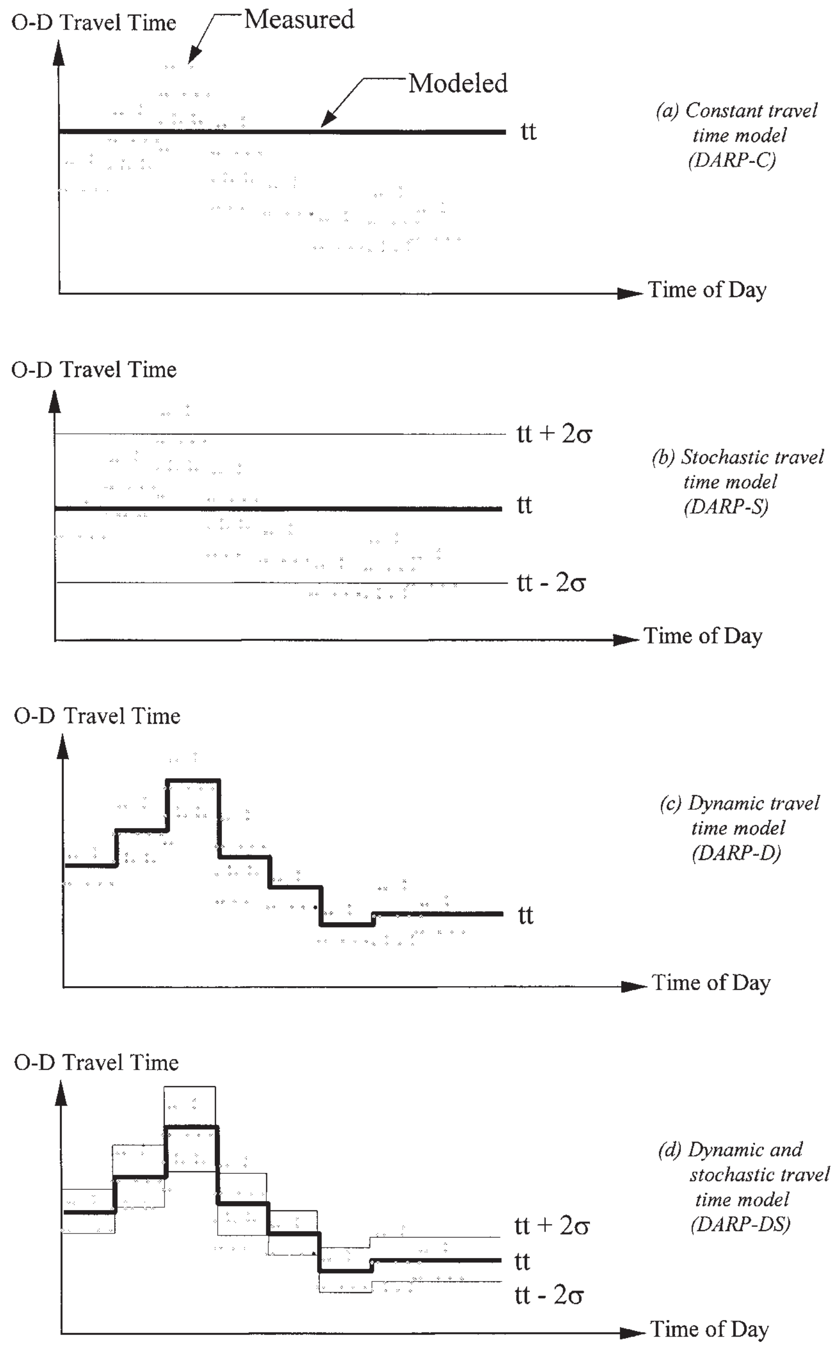

FIGURE 2 Schematic illustration of O-D travel time models.

If all requests are scheduled, then the scheduling process is completed; otherwise, select the next trip $i + 1$ and go to step 3.

Evaluation Approach

To evaluate the potential benefits of considering the dynamic and stochastic variations in O-D travel times, this report concentrates on identifying the difference in the performance of the schedules generated by using the previously described algorithm under different travel time models. The performance of schedules is measured by using the following statistics that are either generated by the scheduling system or obtained through an after-process analysis:

- Total travel time: total duration of all routes excluding the slack time (in hours);
- Vehicle productivity: total number of trips divided by the total travel time (in trips/hour);

- Number of vehicles: number of vehicles scheduled to service all the trips;
- Average ride time: total ride times of the trips divided by the total number of trips (in min);
- Average service time deviation: total service time deviations of the trips divided by the total number of trips (in min);
- Percentage of violated trips: percentage of trips that will not be picked up or dropped off during their service time windows at a given minimum reliability.

EXPERIMENTAL STUDY

A case study was conducted to examine the difference in the scheduling results between considering and not considering the dynamic and stochastic variations of the O-D travel times. The analysis was performed on a real-life instance of the DARP problem, consisting of a weekday morning peak service (7:00 a.m. to 9:00 a.m.) covered by the Disabled Adult Transportation System (DATS) in the city of Edmonton, Alberta, in 1998. The instance includes 463 trips, of which 75 percent were wheelchair trips and the rest were ambulatories. The original trip database did not include the dwell time required at each trip stop; therefore a dwell time of 1 min was added to each pickup and drop-off stop.

A fleet of 60 vehicles was available to provide the service for the morning peak period; the characteristics of these vehicles are summarized in Table 1.

The origin and destination locations of the trips were spread over 515 physical zones covering the municipal area of Edmonton. An asymmetric matrix containing the average travel time during the morning peak period between all the zones was available. This matrix is used as a basis for creating several assumed travel time variation patterns during the morning peak period. Further details along with the computational results are reported in the following section.

It is assumed that the routing and scheduling objective was to minimize the expected total travel time. A maximum ride time of 90 min and a maximum service time deviation of 30 min were used in scheduling to avoid excessive inconvenience to clients.

Effects of Stochastic Variations of Travel Times

For analyzing the impact of the stochastic variations of travel times, two O-D travel time scenarios were considered. As shown in Figure 3, scenario S-1 represents the "true" O-D travel time pattern in the service area, in which the stochastic variations in travel times are modeled by assuming that the travel times between all O-D pairs are independent, normally distributed random variables. The mean travel time for each O-D pair was set to be the same as the average morning peak travel time, given by the original O-D travel time matrix. Because no data are available for use to estimate the true variances of the O-D travel times, it was assumed that the variance to mean ratios (defined as the variance of travel time divided by mean travel time) for all O-D pairs were the same and known. As a result, a matrix containing the variances of travel times between all zones could be calculated by multiplying the mean travel times with the variance to mean ratio. It should be noted that the variance to mean ratio is different from the more commonly used term *coefficient of variation* (defined as the ratio of standard deviation to mean). The variance to mean ratio appears to be more likely to be constant than the coefficient of variation in a given road network, and therefore it was selected for use (*15,16*). Similar to the coefficient of variation, this ratio also reflects the relative variability of travel times in a road network. In the following analysis, variance to mean ratios of 10 sec and 20 sec are used to represent two different levels of travel time variability. Note that the coefficients of variation corresponding to these two ratios are 0.07 and 0.11, respectively, for a trip of 30 min.

TABLE 1 Characteristics of Available Vehicles

Vehicle Type	Quantity	Regular seats	Wheel-chair seats
1	6	4	4
2	17	4	5
3	6	5	4
4	2	6	0
5	8	6	5
6	3	8	4
7	4	8	5
8	3	10	5
9	9	11~16	0
10	2	12	4

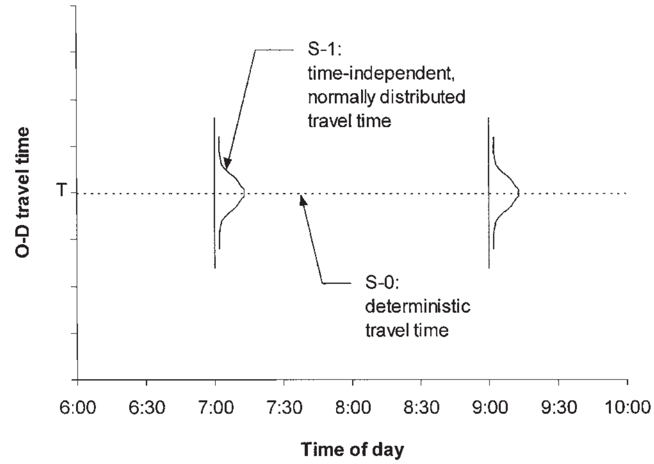

FIGURE 3 O-D travel time scenarios for analyzing effects of stochastic variations in travel times.

Scenario S-0 represents a simplified model of the "true" O-D travel time pattern as represented by S-1. The stochastic variation of travel time is ignored and the travel time between each O-D pair is assumed to be equal to the mean travel time used in S-1. Thus, the original O-D travel time matrix can be used for this scenario.

The morning peak trips in the case problem described previously were first scheduled by using FirstWin with constant travel time (S-0). The travel times between stops and arrival times at individual stops on the obtained schedules were updated by using the stochastic travel times (S-1). The service time constraints subsequently were verified against the probabilistic conditions at a given minimum reliability. The numbers of trips that violated time windows were then determined. Figure 4 shows the percentage of violated trips as a function of the minimum reliability under the two assumed levels of travel time variability. It can be observed that the schedules generated without considering the stochastic variations of travel times (S-0) have a serious reliability problem, even when the variation of travel times is low, as represented by a variance to mean ratio

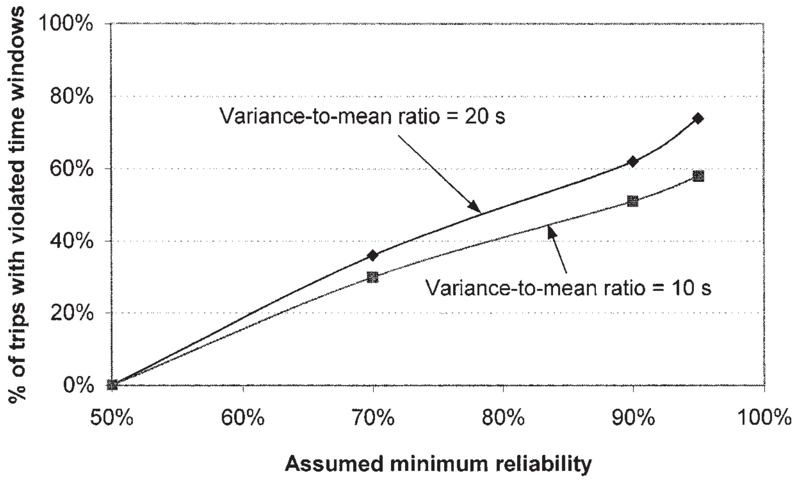

FIGURE 4 Relationship between percentage of violated trips in schedules generated without considering stochastic variations of travel times and minimum reliability under two assumed travel time variability levels.

of 10 sec. For example, approximately 30 percent of trips could not be picked up or dropped off during their desired time windows with a probability of 30 percent (corresponding to the 70 percent minimum reliability level) or higher. The percentage increases to 50 if the assumed minimum reliability is 90 percent. Figure 5 shows that the number of violated trips increases as the variability of travel times increases; nevertheless, the amount of the increase is relatively small compared to the absolute value. This implies that the negative consequences in reliability would be similar at a wide range of travel time variability levels.

If the service reliability is explicitly considered in the scheduling process, the reliability of schedules can then be controlled directly and the percentage of trips with violated time windows can be effectively reduced. Table 2 presents the scheduling statistics based on scenario S-1 with minimum reliabilities of 50 percent, 70 percent, and 90 percent under the two assumed variance to mean ratios. Note that scenario S-1 with a minimum reliability of 50 percent is essentially the same as the deterministic case, i.e., scenario S-0. The use of minimum reliability in the scheduling process, although improving the reliability of the schedules, would cause increased total travel time and reduced productivity, as shown in Table 2. As would be expected, the impact becomes more pronounced as the variability of travel times and the minimum required reliability increase. For example, in the high variability case, the application of a minimum reliability of 90 percent results in a 12.5 percent decline in the vehicle productivity and a 23.4 percent increase in the number of vehicles required.

Table 2 also shows that the rate of decrease in vehicle productivity increases as the minimum reliability increases at both variability levels. The reductions in vehicle productivity caused by the increase

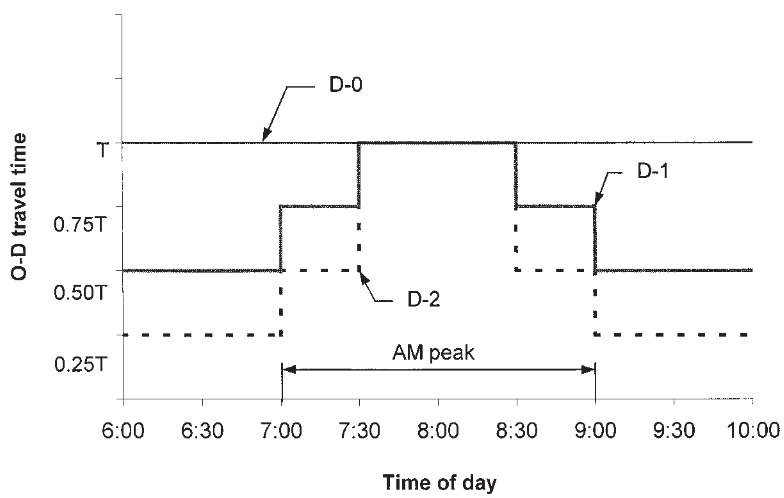

FIGURE 5 O-D travel time scenarios for analyzing effects of dynamic variations in travel times.

TABLE 2 Summary of Solutions with Stochastic Travel Times

Variance-to-Mean Ratio = 20 s					
	50%[a]	70%		90%	
	Total	Total	%[b]	Total	%[b]
Total Travel Time (hr)	90.6	93.5	3.2	101.4	11.9
Productivity (trips/hr)	4.8	4.6	-4.2	4.2	-12.5
No. of Vehicles	47	52	+10.6	58	+23.4
Average Ride Time (min)	42.6	40.3	-5.4	34.6	-18.8
Average Deviation (min)	13.2	12.9	-2.3	13.8	4.5
Variance-to-Mean Ratio = 10 s (travel time is less variable)					
	50%[a]	70%		90%	
	Total	Total	%[b]	Total	%[b]
Total Travel Time (hr)	90.6	91.9	1.4	96.3	6.3
Productivity (trips/hr)	4.8	4.7	-2.1	4.5	-6.3
No. of Vehicles	47	50	+6.4	54	+14.9
Average Ride Time (min)	42.6	39.4	-7.5	37.4	-12.2
Average Deviation (min)	13.2	12.8	-3.0	13.3	0.8

a. In this case, the uncertainty in travel time is not considered.

b. As compared to the case with a minimum reliability of 50%.

of the minimum reliability from 70 percent to 90 percent are more than twice as much as those from 50 percent to 70 percent. The implication of this result is that one should be cautious in using high reliability levels in the scheduling process because it may cause a significant amount of decrease in vehicle productivity.

As it would be expected, applying the minimum reliability criteria in the scheduling process yields schedules with improved quality of service, as indicated by lower average ride time and service time deviation. It is also interesting to observe that the decrease in the productivity and the increase in the number of vehicles required are much less significant in the situation in which the travel time variability is not very high and a moderate level of reliability is required. For example, the application of a minimum reliability of 70 percent results in a 2.1 percent decrease in the vehicle productivity and a 6.4 percent increase in the number of vehicles.

Finally, it should be noted that although these findings were obtained by considering the variability of travel time, they should be equally valid for the impact of the variability of dwell time.

Effects of Dynamic Variations of Travel Times

The impact of the dynamic variations of travel times on the reliability of schedules is investigated by using the O-D travel time scenarios D-0, D-1, and D-2, as illustrated in Figure 5. Scenarios D-1 and D-2 exemplify the "true" O-D travel time patterns, in which dynamic variations of travel time are explicitly considered. The scenario D-1 represents the situations with a relatively smooth temporal variation, and D-2 represents the situations with a relatively peaked temporal variation. Both scenarios are assumed to be peaked at the time interval from 7:30 to 8:30 with peak travel time for each O-D pair equal to the average morning peak travel times for the O-D pair. The travel times for other time intervals are lowered, following the scales shown in Figure 5.

Scenario D-0 represents the simplified model of the true O-D travel time pattern as represented by D-1 and D-2. The travel times between all O-D pairs are assumed to be time independent, and thus the dynamic variations are ignored. The travel time between each O-D pair is assumed to be the same as the travel time during the peak time interval in scenario D-1 and D-2 (from 7:30 to 8:30, as shown in Figure 5), and therefore it is given by the original O-D travel time matrix. This assumption reflects the conservative approach that would be taken by most schedulers in the situations in which a single O-D travel time is to be used.

Table 3 lists the scheduling statistics under those O-D travel time scenarios. As would be expected, the generated routes are less productive with the static travel time case, because the travel times used in this case are larger than the true travel times, as represented by D-1 and D-2. Three additional vehicles are scheduled as compared to scenario D-1, and seven additional vehicles as compared to scenario D-2.

To reveal the problem caused by not considering the dynamic variation of travel times, schedules obtained by using the static O-D travel time (D-0) are recalculated with the assumed true O-D

TABLE 3 Summary of Solutions with Three Dynamic O-D Travel Time Scenarios

	O-D Travel Time Scenario		
	D-0	D-1	D-2
Total Travel Time (hr)	104	91.3	69.8
Productivity (trips/hr)	4.1	4.7	6.2
No. of vehicles Required	44	41	37
Average Ride Time (min)	42.3	39.8	36.1
Average Deviation (min)	13.2	13.9	12.6

travel times as represented by D-1 or D-2. Schedule performance statistics, such as the total vehicle hours, the vehicle productivity, and the percentage of violated trips at three violation levels, are then determined. Table 4 gives these statistics under the true O-D travel time scenarios.

It can be observed from Table 4 that ignoring the dynamic pattern of travel times would result in routes with a significant number of trips that would not be served within their desired service time windows. In scenario D-2, 35 percent of the trips would violate their service time windows by more than 5 min. When the O-D peaking is smoother, as seen in scenario D-1, only 14 percent of trips would violate their service time windows by more than 5 min. The problem is also revealed in the maximum time violation of the desired service time windows: 39 min in the case of peaked temporal variation and 19 min in the case of smooth temporal variation.

Apart from the reliability problem, ignoring the dynamic travel time pattern in scheduling would result in inferior schedules in terms of vehicle productivity. For example, the vehicle productivity will increase approximately 23 percent (from 4.55 to 4.70) if scenario D-1, representing the true dynamic pattern, is used in the scheduling process.

CONCLUSIONS

A series of numerical experiments was performed on a real-life paratransit scheduling problem under several hypothetical travel time variation models. The intent was to examine the potential effects of travel time variations on the quality of service and productivity of a paratransit system. A paratransit vehicle routing and scheduling system capable of solving the DARP with various O-D travel time models was used in the experiments. The following findings were obtained:

1. Routing and scheduling without considering the stochastic variations of travel times would yield schedules with a high percentage of trips that may not be picked up or dropped off during their desired time windows.
2. Explicit consideration of the variability of travel times could effectively improve the reliability of the schedules generated by the scheduling process. However, the improvement in reliability would also result in decreased productivity and increased number of vehicles required. The decline in system productivity is marginal when the travel time variability is low (e.g., with a variance to mean ratio of less than 20 sec in the presented case) and the required reliability is moderate (e.g., around 70 percent).
3. The study clearly shows that there are several serious negative consequences from ignoring the dynamic variation of O-D travel times in the scheduling process, including declined vehicle productivity, increased number of vehicles required, and high percentage of trips that may not be served during their desired time windows. These negative effects suggest the possible gains if the true dynamic variation can be taken into account in the scheduling process.
4. The implied benefits of considering the dynamic and stochastic variations of travel times in the scheduling process are increased customer satisfaction and reduced system operating cost.

Finally, it should be noted that these results were obtained from the analysis of a specific case with hypothesized O-D travel time variation patterns and therefore preclude definite generalizations. Nevertheless, they offer a meaningful illustration of the issues that paratransit service providers should consider, while suggesting the need for further research. Future research will involve investigations on more extensive cases with more realistic system settings and O-D travel time data. Another important research direction is to investigate the potential of using real-time data, such as current vehicle locations and probe travel times, to improve the productivity and reliability of a paratransit system.

TABLE 4 Schedule Statistics for D-0 with "True" O-D Travel Times

	Assumed "True" O-D Travel Time Scenario	
	D-1	D-2
Total Travel Time (hr)	95.8	85.1
Productivity (trips/hr)	4.6	5.1
% of violation at:		
0~5 min	12.8	13.8
6~10 min	11.2	15.6
>10	2.8	19.5
Maximum of violation (min)	19.0	39.0

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